Athemath Fall 2022 Admissions Quiz

Athemath Staff

Due September 3rd, 2022

§1 Instructions

For all of the problems below, **proof-based solutions are encouraged**. We would like you to explain all of your steps, instead of just giving an answer. If you don't have experience with proofs, just try to explain your answer as much as you can. LaTeX submissions and *neat*, dark handwriting submissions are both allowed.

We also encourage you to try the entire test. While later problems will generally be harder, they also play to different strengths and you may find one particularly easy.

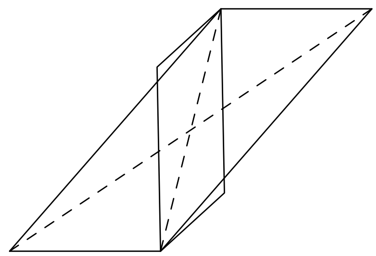
Please do not use computer programs, Google, WolframAlpha, GeoGebra, etc. to help you find solutions. Additionally, please do not discuss this quiz with anyone else until after the application deadline has passed. **If you find the test difficult, that's because it's designed to be.** If you get stuck, take a walk, try a different problem, or try a strategy you dismissed at first. And remember that you don't have to solve all—or even a majority—of the problems to get in. Historically, the average admitted student solves around two problems.

Ask for clarifications by emailing Serena at serena.an@athemath.org. Submit your completed solutions to the application Google Form by **September 3rd, 11:59PM Eastern**. As a reminder, only students of underrepresented genders can apply. Have fun!

§ 2 The Problems!

Problem 1

Let P_0 be the parallelogram with vertices $(0,0),(5,0),(12,h),(7,h)$. A parallelogram P_1 similar to P_0 is drawn so that its long diagonal coincides with the short diagonal of P_0. Given that the area of P_1 is one-third that of P_0, what is h? (Note: *figure shown not to scale.*)

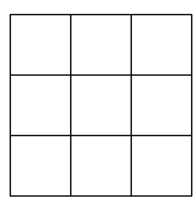


Problem 2

The fraction $\frac{7}{12}$ has the interesting property that it is the largest fraction that can be written as the unordered sum of two distinct unit fractions in two different ways; namely, $\frac{7}{12} = \frac{1}{2} + \frac{1}{12} = \frac{1}{3} + \frac{1}{4}$. What is the second largest fraction with this property?

Problem 3

Is it possible to place the nine letters G, I, R, L, S, T, O, E, and H in a 3×3 grid such that consecutive letters in each of the words 'GIRLS' and 'TOGETHER' are in cells that share a side?



Problem 4

Triangle ABC has side lengths $AB = 6$ and $AC = 8$. Let D be the second intersection of the angle bisector of $\angle BAC$ with the circumcircle of ABC, and let E be the foot from D to AC. Compute AE.

Problem 5

Let $n \geq 2$ be an integer. A permutation of n ones, $n-1$ twos, …, one n is *stable* if the k^{th} instance of a precedes the k^{th} instance of $a+1$ for all positive integers k and a. Prove that the number of stable sequences is even.